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STUDY OF CORRECTIONS TO OBTAIN THE RIGOROUS ORTHOMETRIC HEIGHT

F.L. Albarici[1], I. Foroughi[2], M.C. Santos[3], G.N. Guimarães[4], J.L.A. Trabanco[5], R. Kingdon[6], P. Vaniček[7]

[1] IFSULDEMINAS – Campus Inconfidentes, Brazil
[2,3,6,7] GGE - University of New Brunswick, Canada
[4] UFU - Campus Monte Carmelo, Brazil
[5] FEC/UNICAMP, Brazil

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ABSTRACT

The height system in Brazil is defined as normal-orthometric height. According to IBGE, (2011) only the non-parallelism corrections of equipotential surfaces were applied to GNSS/leveling network of Brazil. The objective of this study is to first calculate the Helmert orthometric heights of the GNSS/Leveling points of the city of Campinas/SP, and then convert them to rigorous orthometric heights. To do this, the geopotential numbers were calculated for all points and used to compute normal and Helmert orthometric heights. For converting Helmert to rigorous orthometric heights, the corrections which were taken into account are: (1) Second-order correction for normal gravity; (2) Second-order correction for the Bouguer shell; (3) The geoid-generated gravity disturbance; (4) The terrain/roughness-generated gravity; (5) The lateral variation of topographical mass–density. These corrections were calculated individually; corrections (1) and (2) were insignificant, and the total corrections of (3), (4) and (5) were in order of mm level. The range of differences between normal and rigorous orthometric heights were in the vicinity of 5cm on mean.

Keywords: Rigorous Orhometric Heights, Orthometric Height, Gravity

1- INTRODUCTION

Mostly in South America, the height systems have problem in many countries, and over the last decades these countries have been making efforts to modernize their heights system. The height system in these countries were defined as normal or orthometric which was the best solution for the altimetric system. In this context, the height reference system of Brazil is characterized in normal-orthometric system; according to IBGE, (2011) the only correction applied to equipotential surfaces were non-parallelism corrections.

The observed levelling data (raw) were adjusted and corrected from the term applied to the observed slopes (Eq. (1)), which reduces the error caused by the non-parallelism of the equipotential surfaces, this correction is performed by the lack of available gravimetric data (Pina et al., 2006):

$$-\frac{H_m(C_1\sin 2\,\varphi_m+2C_2\sin 4\varphi_m)\Delta_\varphi}{(1+C_1 sin^2\varphi_m+C_2 sin^2 2\varphi_m)} \qquad (1)$$

Where, H_m is the mean height of the levelling section; φ_m is section's latitude; Δ_φ represents the difference between latitudes of the section; $_1$ and $_2$ are the coefficients of the normal gravity field, where the values are 0.0053023655 and -0.0000059, respectively. This type of heigts replaces comlpetely the use of the potential of the real Earth (W), by the use of the esferopotential (U). Thus, the numbers of the geopotential (C) are replaced by the numbers of the normal potential (C^N) (Filmer et al., 2010). Height is still a challenging problem in Geodesy. According to Blitzkow et al. (2007), the concept of heights is related to the geodesy boundary-value problem (GBVP). Consequently, the question needs to be thought not only by the geometrical point of view, but, especially by a physical one.

The objective of this study is to calculate the corrections in Helmert orthometric heights and carry out its updating to the rigorous orthometric height. The effects of these corrections will be applied in the city of Campinas/SP, where the enough observations for the accomplishment of these calculations are available.

2- REVIEW OF THE THEORY

According to Kingdon, 2005, the geopotential number is defined as the difference between the potential on the geoid (W_0) and the potential at topography (W) (Eq. (2)). The geopotetnial number of a points is also defined as a function of gravity (g) from the geoid to the point at topography (Eq. (3)):

$$W_0 - W \tag{2}$$

$$\int_O^{H_A} g\ dH \tag{3}$$

Equation (3) can be written as:

$$\bar{g} H_A \tag{4}$$

It means that the geopotential number is equal to a orthometric height multiplied by the mean acceleration of gravity ($\bar{g}$) along the plumb line (Meyer et al, 2006). Depending on the defintion of the mean gravity along plumbline, various types of elevation can be realized. Mathematically, the orthometric height, H^0 of a point is defined as the geopotential number (C_P) divided by the mean gravity along the plumb line (g_m) between the point of interest and the geoid (Kingdon et al, 2005).

$$H^0 \quad \frac{C_P}{g_m} \tag{5}$$

Where: $g_m \quad \frac{1}{H^0} \int_{P_0}^{P} g\ dH$ (6)

According to Eq. (5) one cannot guarantee that the points with same orthometric heigts are on the same equpotential surface as they depend also on the mean gravity along the plumbline. However, the acceleration of gravity depends on heights, latitude, and mass distribution to be reason of concern. There is no reason why the average gravity is equal and, in fact, it is normally not. Therefore, two points of the same orthometric heights do not need to have the same potential energy density, which means that they do not need to be on the same potential surface and therefore not the same heights from the perspective of geopotential numbers (Meyer et al. , 2006). The separation between the geoid and the quasi-geoid is important in the context of the modernization in Brazilian heights system. This separation reflects the difference between orthometric and normal heigth. Regardless of the adopted height system by neighbour countries, it is possible to convert the unify the height system between them with enough accuracy. Therefore it is necessary to model the accurate transformation between normal and orthometric heigts, although there may be in the future a unequivocal reference to the geopotential numbers (Ferreira et al, 2011). For definition of he normal height, according to Hofmann-Wellenhof and Moritz, 2006, for the point that the gravity field of the Earth becomes normal, ie, W = U, g = γ, T = 0, the normal heights are defined by:

$$\int_0^{H^N} \gamma\ dH^N \tag{7}$$

$$\bar{\gamma}\ H^N \tag{8}$$

Where H^N is the normal height, γ is the normal gravity, T is the disturbing potential. Equations 7 and 8 are similar to orthometric height equations, but they have totally different meaning. The zero used in the lower integral is for the reference on the ellipsoid, so the normal height will depend of the choice of the ellipsoid and the datum. Normal gravity is an analytical function whose mean can be calculated, but no gravity observation is required. Finally, from its definition one finds that the normal height (H^N) is the altitude of the ellipsoid where the normal gravity potential is equal to the real geopotential of the point of interest (Meyer et al, 2006).

According to Vaníček et al. (2003), the orthometric heights and the normal heights can be obtained accurately. Foroughi et al. (2017) showed that that rigorous orthometric heihgts can be defiend using freely availabe data-sets and with high accuracy. Corrections of converting Helmert heigths to rigorous orthometric heights are summerized in Santos et al. (2006) and Foroughi et al. (2017). The main problem with the rigorous definition of orthometric height is computing the the Earth mean gravity value along the plumb line between geoid and topography. In order to find the exact relationship between the rigorous orthometric heights and the normal heights of Molodensky, the mean gravity is decomposed into: mean normal gravity, the mean gravity values generated by the topographic and atmospheric masses and the perturbation of the average gravity generated by the masses contained in the geoid. The mean normal gravity is evaluated according to the Somiglian-Pizzetti theory of the normal gravity field generated by the ellipsoid of revolution. Using the Bruns theorem, the mean values of gravity along the plumb line generated by the topographic and atmospheric masses can be computed as the integral mean between the Earth's surface and the geoid. As the disturbing gravitational potential generated by the masses within the geoid is harmonic above the geoid, the mean value of the gravitational perturbation generated by the geoid is defined by the application of the Poisson integral equation (Tenzer et al, 2005).

The five corrections that are applied to Helmert heigths are: Second-order correction for normal gravity; Second-order correction for the Bouguer shell; The geoid-generated gravity disturbance;The terrain/roughness-generated gravity; The lateral variation of topographical mass–density (Santos et al, 2006).

(1) Second-order correction for normal gravity

$$\gamma\left(\frac{H^o(\Omega)}{r_t(\Omega)}\right)^2 \approx \gamma\left(\frac{H^o(\Omega)}{a}\right)^2 \quad (9)$$

(2) Second-order correction for the Bouguer Shell

$$\frac{4}{3}\pi G\rho_0 \frac{H^o(\Omega)^2}{R+H^o(\Omega)}\left(2-\frac{H^o(\Omega)}{R+H^o(\Omega)}\right) \quad (10)$$

(3) The geoid-generated gravity disturbance

$$\left[\overline{\delta g}^{NT}(\Omega) - \delta_g^{NT}(r_t,\Omega)\right] \quad (11)$$

(4) The terrain/roughness-generated gravity

$$\bar{g}_R^T(\Omega) - g_R^T(r_t,\Omega) \quad (12)$$

(5) The lateral variation of topographical mass-density

$$\left[\bar{g}^{\delta_\rho}(\Omega) - g^{\delta_\rho}(r_t,\Omega)\right] \quad (13)$$

Where:
a is the major semi-axis of the reference ellipsoid;
G Newton's gravitational constant;
$r_t(\Omega)$ geocentric radius of the Earth's;
R is the inner radius of the shell;
$H^o(\Omega)$ orthometric height;
$\overline{\delta g}^{NT}$ mean gravity disturbance generated by the geoid;
$\delta_g^{NT}(r_t,\Omega)$ Gravity generated by masses contained within the geoid;
$\bar{g}_R^T(\Omega)$ mean value Gravitation generated by the terrain roughness;
$g_R^T(r_t,\Omega)$ Gravitation generated by the terrain roughness;
$g^{\delta_\rho}(r_t,\Omega)$ Effect on gravitation due to lateral mass–density variations inside the topography with respect to the reference value of ρ_0 = 2,670 kgm^{-3};
ε^{H^o} Correction to Helmert's orthometric height to convert it to the rigorous orthometric height (Tenzer et al. 2005)

Converting the Helmert heigths to rigorous orthometric heigths makes the evaluation of geoid with GNSS/Leveling points more accurate. Finally, the total correction to the Helmert orthometric height (Santos et al, 2006).

$$\varepsilon^{H^o} \quad (1+2+3+4+5) \quad (14)$$

3- DATA SET AND NUMERICAL TEST

Data from the Campinas city were obtained from UNICAMP (University of Campinas). The study area is located between latitudes -23 and -22 and longitudes -48 and -46 and contains 40 terrestrial gravity observations and 40 GNSS/levelling points in this area. The GNSS/levelling point data set contains, apart from the horizontal and vertical location information, where the geodetic height, derived from GNSS observation, and normal height, derived from spitit levelling. Figure 1 shows the location of the area:

Fig. 1 – Study Area

The number of the geopotential was calculated by equation 3, in this way one can obtain the normal heights and Helmert's orthometric height, and apply the five corrections (Eq. 9, 10, 11, 12 and 13) to the rigorous orthometric height.

The total corrections to Helmert's orthometric height are showed in Figure 2, and their statistics are provided in table 1.

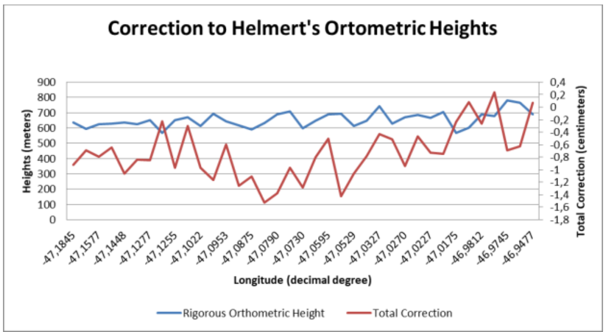

Fig. 2 – The total correction to Helmert's orthometric height

TABLE 1 – THE STATISTICS OF CORRECTION TO HELMERT'S ORTHOMETRIC HEIGTS

Orthometric Heights Correction Term	1 (mm)	2 (mm)	3 (mm)	4 (m)	5 (m)
min	-0.012	-0.011	-1.245	-0.0097	-0.0128
max	-0.005	-0.004	2.622	0.0008	0.0027
Mean	-0.0071	-0.0070	1.461	-0.0017	-0.0057
STD	0.0017	0.0017	0.9971	0.0020	0.0042

As expected, the higher the point height, the greater the correction values. The second-order correction for normal gravity (Term 1) and second-order Correction for the Bouguer Shell (Term 2) are directly correlated with topography, and do not need to be used in practice because both are very small. The non topographic correction (Term 3) has a direct correction with the terrain, in this case, and the relation of the terrain generated (Term 4) and the lateral density (Term 5) with the terrain is very difficult to explain. The corrections terms to Helmert's orthometric Heights is presented by the figure 4.

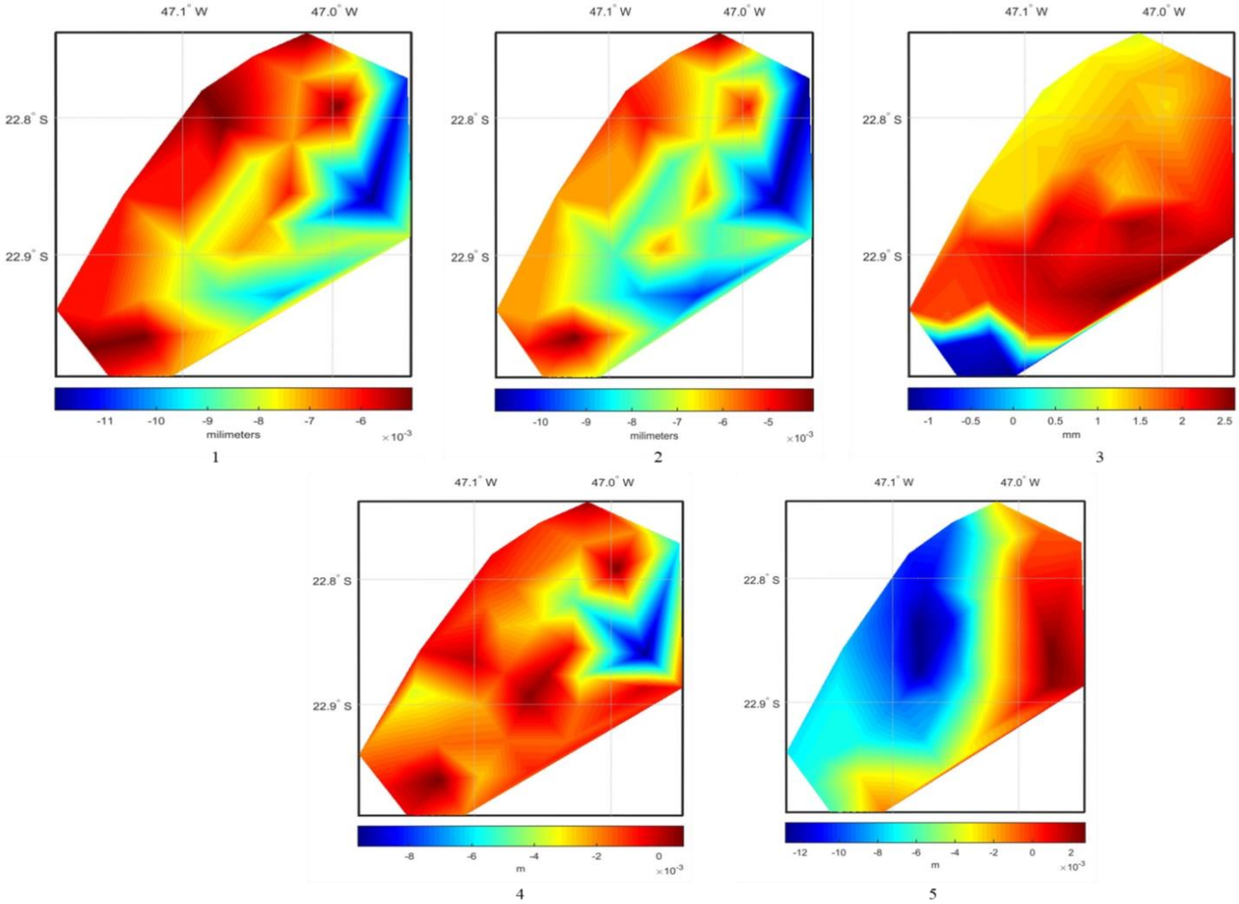

Fig. 4 – The correction terms to Helmert's orthometric height to get orthometric height.

The results show that terms 1 and 2 have a small magnitude and in practice their results can be neglected, where statistically their minimum, maximum, mean and standard deviation are respectively: -0.012 mm; -0.005 mm; 0.0071 mm; 0.0017 m and -0.011 mm; -0.004 mm; -0.0070 mm; 0.0017 mm. The 3th corrective term has the highest correction values and these values have large correlation with the terrain. The highest values of the corrections are associated to the highest heigths, having minimum and maximum values of -1.245 mm and 2.622 mm, mean 1.461 mm and deviation standard of 0.9971 mm. The 4th term usually corresponds to the second major correction, but specifically in this case, does not have a large variation in the terrain, with the minimum and maximum values being -0.0097 m; 0.0008 m, with mean and standard deviation at -0.0017 m and 0.0020 m, respectively. And finally the term 5 which is the third largest correction with a minimum of -0.0128 m, maximum of 0.0027 m, average of -0.0057 m and standard deviation 0.0042 m.

The difference between the rigorous orthometric height and normal heights is shown in figure 5.

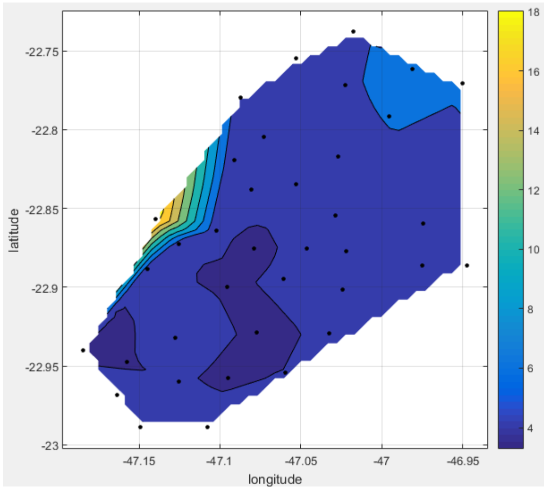

Fig. 5 – Difference between the Rigorous Orthometric Height and Normal Height (units in centimeters)

The statistics of the residuals between computed rigorous orthometric height and normal heights are summarized in table 2.

TABLE 2 – THE STATISTICS OF RESIDUAL (NORMAL AND RIGOROUS ORTHOMETRIC HEIGHT).

Statistics	Min (cm)	Max (cm)	Mean (cm)
Differences	3,22	19,55	5,058

4- CONCLUSION

Based on the numerical analysis performed in points distributed around the city of Campinas/SP, it can be concluded that the largest and smallest corrections applied to the Helmert's heights were 1.52 and 0.23 centimeters. These corrections were calculated individually where corrections (1) and (2) have no contribution because they are very small, but corrections (3), (4) and (5) have the greatest contribution in obtaining the rigorous orthometric height.

About the differences between normal and rigorous heights, it turns out that these differences have an mean correction of 5 cm which in scientific terms is a considerable difference.

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